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1989 J. Phys.: Condens. Matter 1 7033

(http://iopscience.iop.org/0953-8984/1/39/014)

J. Phys.: Condens. Matter **1** (1989) 7033–7036. Printed in the UK

On the determination of the resonant tunnelling time

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Received 22 February 1989

Abstract. The one-dimensional resonant tunnelling problem has been investigated using the method of expansion in terms of the eigenfunctions of the scattering problem. It is shown that tunnelling is in fact described by two different times, and analysis is performed of the distortion of the tunnelling pulse.

Recently, Collins *et al* (1987a, b) have performed a detailed study of the resonant tunnelling process. Both analytically and numerically, they have calculated the electron tunnelling time which had been associated with the time necessary for the maximum of the electron wave packet to pass through the two-barrier (or similar) potential structure. However, the packet form changes as a result of tunnelling; so the single time cannot properly describe the tunnelling of the whole packet. It will be shown in this paper that the full dynamical description of the wave pulse resonant tunnelling is given by two times.

When an electron pulse collides with a scatterer, an interaction influences its movement. As a consequence a time delay appears between the scattered packet and the freely moving wave packet which covers the same path without interaction (Wigner 1955). The time delay can be expressed as (Goldberger and Watson 1964)

$$Q = -\mathrm{i}\,\mathrm{d}(\ln \mathbf{S})/\mathrm{d}\,\varepsilon \tag{1}$$

where $\mathbf{S}$ is the corresponding scattering matrix.

Two points should be mentioned in connection with this expression. First, equation (1) is valid for a packet having a well defined energy; the uncertainty $\Delta\varepsilon$ should be small compared with the width γ of the resonant state or, more generally, with the width of the energy interval where the scattering matrix changes significantly. Secondly, it is clear from (1) that the time delay has a definite value only for the eigenfunctions of the scattering problem because in that representation the S-matrix has a diagonal form. Every eigenfunction has its own delay time, and the resonance at a given energy for some function affects only its delay while the other delay times are still due to the potential scattering.

We shall use this approach to analyse the one-dimensional problem of resonant tunnelling through a symmetrical potential structure. The asymptotic expression for the electron wavefunction can be written in the general form (Landau and Lifshitz 1974)

$$\psi(x) = F(-n)\exp(-\mathrm{i}kxn) - \exp(\mathrm{i}kxn)\sum_{m=\pm 1} S_{nm}F(m) \tag{2}$$

where $n = \operatorname{sgn} x = \pm 1$, S_{nm} are the elements of the S-matrix for scattering by the potential

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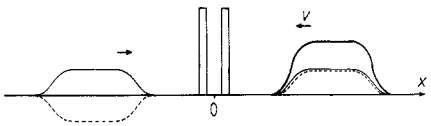

Figure 1. Expansion of wave packet (——) in terms of S (——) and AS (----) pulses before collision with the two-barrier potential structure. The relationships of the wave phases in the pulses are denoted by the signs of their envelopes.

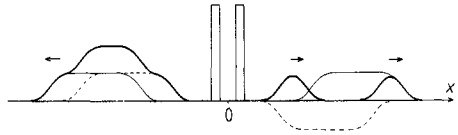

Figure 2. The result of resonant tunnelling of the packet through the structure. The resonant AS component is behind the non-resonant S component. The delay has been increased in the figure to make it more apparent. The change in the AS waves phase after collision is denoted by the change of the AS packet envelope sign in comparison with figure 1.

structure and $F(\pm 1)$ are arbitrary coefficients. For a symmetrical potential, one can choose as the basis set the odd and even functions (2) with $F_s(n) = F_s(-n)$ and $F_a = -F_a(-n)$; correspondingly:

$$\begin{aligned} \psi_s &= \exp(-ik|x|) - S_s \exp(ik|x|) \\ \psi_a &= [\exp(-ik|x|) - S_a \exp(ik|x|)] \operatorname{sgn} x. \end{aligned} \tag{3}$$

In this representation the S-matrix has a diagonal form with the two eigenvalues $S_s = \exp(2i\delta_s)$ and $S_a = \exp(2i\delta_a)$, δ_s and δ_a being the phase shifts.

A quasi-local resonant state in a symmetrical potential can be either odd (AS) or even (S). For example let us consider tunnelling at energies near the resonant energy for odd states. In this case the phase shift for the AS waves has the standard resonant form (Landau and Lifshitz 1974)

$$\delta_a = \delta^{(0)} - \tan^{-1}\{\gamma/[2(\varepsilon - \varepsilon_0)]\} \tag{4}$$

where ε_0 and γ denote the energy and the width of the resonance ($\varepsilon_0 \ll \gamma$), and $\delta_a^{(0)}$ is the phase shift due to the potential scattering. $\delta_a^{(0)}$ is a slowly varying function of energy so that one can neglect its variation in the vicinity of resonance and treat $\delta_a^{(0)}$ as a constant. The symmetrical wave phase shift δ_s at $\varepsilon \simeq \varepsilon_0$ is also caused by potential scattering; hence one can also put $\delta_s = \delta_s^{(0)} = \text{constant}$.

Usually, resonant tunnelling is studied for the potentials which are almost non-transparent to electrons having energies far from the resonant values. In this case, one obtains

$$\delta_a^{(0)} = \delta_s^{(0)}. \tag{5}$$

To demonstrate this, let us consider the wavefunction $\Psi = (\psi_a + \psi_s)/2$. This function does not contain the wave from $-\infty$, and so the coefficient of $\exp(-ikx)$ at $x < 0$ is the transmission coefficient t:

$$t = (S_a - S_s)/2. \tag{6}$$

If for energy sufficiently far from resonance the barrier is not transparent, i.e. $t = 0$, and if the resonant phase shift is small, one obtains $S_a = S_s$ or $\delta_a^{(0)} = \delta_s^{(0)}$.

Now let us consider the resonant tunnelling of the wave packet with $\varepsilon = \varepsilon_0$ which moves towards the potential structure from $+\infty$. The pulse can be expanded in terms of eigenfunctions (3), i.e. expressed as a sum of the odd and even parts (figure 1). Each component has its own specific phase shift and time delay; hence the pulse tunnelling is characterised by two different times, one of which is resonant and the other is not.

Before collision with the structure the waves in both S and AS packets are in phase so one adds to another at $x > 0$ to form the initial pulse and at $x < 0$ they cancel one another. After the collision the waves in the AS pulse undergo a resonant phase shift in addition to that caused by potential scattering so that S and AS waves become displaced in phase by $2(\delta_s - \delta_a) = \pi$ for $\varepsilon = \varepsilon_0$ (see (4) and (5)). Consequently the AS packet now adds to the S packet at $x < 0$ to form the tunnelling pulse whereas at $x > 0$ the packets cancel each other. However, the compensation at positive x is not complete because these two components have different delays; for the resonant pulse

$$Q_a = 2\,\mathrm{d}\delta_a/\mathrm{d}\varepsilon \simeq 2(\mathrm{d}/\mathrm{d}\varepsilon)\{\tan^{-1}[\gamma/2(\varepsilon - \varepsilon_0)]\} = \gamma/[(\varepsilon - \varepsilon_0)^2 + \gamma^2/4] = 4/\gamma \qquad (7)$$

and for the non-resonant pulse the delay is much smaller, i.e.

$$Q_s = 2\mathrm{d}\,\delta_s/\mathrm{d}\,\varepsilon \ll Q_a(\varepsilon_0)$$

because the potential scattering phase changes significantly only for energy variation of order of $\varepsilon_0 \gg \gamma$. Hence two reflected packets appear, as has been found previously by numerical calculations (Collins *et al* 1987b). The difference in delay values also leads to distortion of the tunnelling pulse (figure 2).

The leading edge of the tunnelling packet coincides with the front of its non-resonant component (which is even in our example). Its delay is small; the absolute value of Q_s is of the order of magnitude of the electron flight time through the potential structure. Strictly speaking, Q_s is negative so that it is not a decay but rather an advance† but the important point is that the leading edge of the tunnelling packet shows no resonant delay. On the contrary, the trailing edge of the tunnelling pulse coincides with that of the resonant component which has a resonant delay Q_a (see (7)). Q_a is determined by the lifetime of the quasi-local state in the potential structure and can be large (Ricco and Azbel 1984). The mean delay time, i.e. the delay τ of the pulse middle point (the pulse's maximum) equals $(Q_a - Q_s)/2 \simeq Q_a/2$. This agrees with the result obtained earlier (Collins *et al* 1987a, Teranishi *et al* 1987): $\tau = \mathrm{d}(\arg t)/\mathrm{d}\,\varepsilon$, where t is the transmission coefficient. Using (6), one easily obtains $Q_a/2 = \tau$.

As a result of the resonant tunnelling the length of electron packet increases by $v(Q_a - Q_s) \simeq vQ_a$, where v is the group velocity and Q_a the resonant delay (7). The leading edge of the packet passes through the barrier almost without delay and the pulse elongates mainly owing to the delay of the trailing edge. It is worth noting that for the packet with energy width $\Delta\varepsilon \ll \gamma$ the elongation caused by the tunnelling is small in comparison with the pulse length.

To conclude, the one-dimensional resonant tunnelling problem has been investigated here using the method of expansion in terms of the scattering problem eigenfunctions. It is shown that the wave pulse tunnelling is described by two different times. Analysis of the tunnelling pulse distortion has been performed.

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† This is connected with the fact that the potential of the two-barrier structure is repulsive for the potential (non-resonant) scattering. So the non-resonant wave packet is reflected from the potential barrier and does not penetrate to the coordinate origin as the freely moving wave packet would do. Hence the reflected packet's path is shorter and the pulse leads the free wave packet. This result is evident for simple models which can be treated analytically (Flugge 1971).

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